A system for online assessement of mathematical knowledge

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Abstract
The paper presents a brief review of a system for online testing of mathematical competences and skills. The system has enhanced testing abilities. It allows the teacher to define a problem by using dynamic mathematical models. Also the system automatically evaluates five different types of answers which make the evaluation process faster and more precise.

Keywords: dynamic model, automatic evaluation, mathematic, knowledge

1 Introduction

Online automated assessment of mathematical knowledge poses two main problems: (i) an adequate content representation of questions and tasks, and (ii) an automated evaluation of the answers. The assessment system to be discussed here below was designed and developed in accordance with the requirements defined by a team of researchers from the Institute of Mathematics and Informatics of Bulgarian Academy of Science involved in the implementation of the Inquiry based mathematics education [1, 2]. The most essential user requirements have been determined as follows:

- Enhanced content representation with dynamic interactive models by means of Geogebra [3]
- Starting tests by time scheduler
- Real time recording of the answers
- Separate storage of conditions and answers
- Encrypting the correct answers
- Automated evaluation of the results
- Integration within the CMS Invision Power Board

Although most of these functionalities are quite common for such type of testing systems there are some new extensions such as including of dynamic models and automation of the evaluation process.

When working on mathematical problems the students experience variety of difficulties such as inability to easily connect abstract or conceptual aspects of math with reality or inability to effectively visualize math concept [4]:

An attempt to ease the transition between real world perception and its mind replication is to enhance formal and verbal representation of mathematical tasks with dynamic and interactive models [1]. A dynamic and interactive model is a visual illustration of a mathematical object which updates its expression values upon modification.

When numerous users pass online tests the only feasible solution to check the results and grade them is the automated evaluation. The evaluation system should recognize not only everyday used answers such as *a single choice – a single answer* but also answers which are more specific to mathematic free numerical answers mapped by some valuation function.

2 Implementation

The assessment system consists of four main parts:

- Representative
- Communicative
- Data warehouse
- Scheduler
- Analytic

The representative part is responsible for interpretation of the questionnaires. Formally all questions and their valuations are described by using XML schema. The correct answers are encrypted and stored in separate data warehouse. This is done for security reasons to protect simultaneous access to questions and answers. The communication module carries synchronous duplex data exchange between user agent and the server. The data flow relies on AJAX technology. The user is acknowledged if its answers are not accepted by the system for any reason. Analytical part evaluates an individual test in real time or by the administrator demand. The overall results are reported for all users, tests and individual questions. The scheduler allows tests to be started in arbitrary time window for any group of users.

The system architecture building blocks are shown in Fig. 1.

3 Automatic evaluation of the answers

The system interpreter currently is able to recognize five types of questions and their answers:

- Single choice – single answer
- Single choice – multiple answers
- Multiple choice – multiple answers

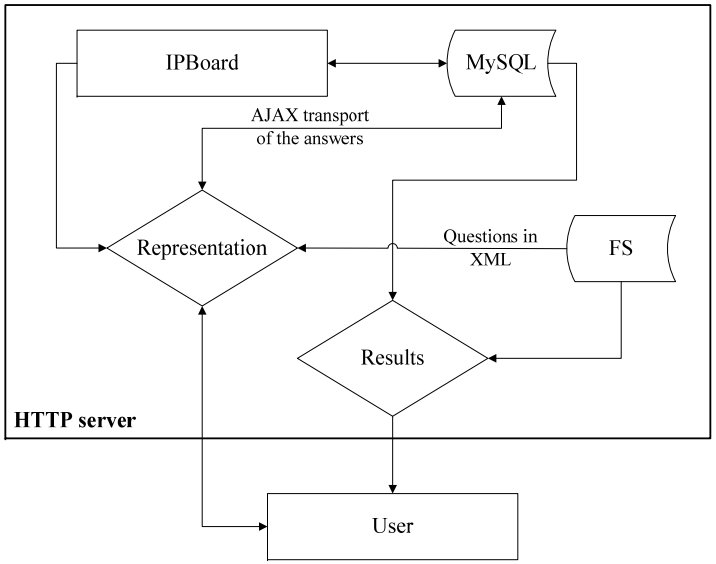


Fig. 3. *The scheme depicts flowchart of data circulating among the system components. The questions and valuation of the answers in XML format are stored in file system FS. Correct answers and scheduling information are stored in MySQL database. User information is retrieved from CMS Invision Power Board [5]. The two decision blocks which drive whole application are* ***Representation*** *and* ***Results****.*

- Exact number
- A number in discrete intervals

The evaluation function f in a *single choice – single answer* question type is defined as:

$$f: A \to \{0, n\}$$
$$where\ A = \{\, x \in \mathbb{N} \mid x \leq 9\}\ and\ n \in \mathbb{R}$$

This function is total surjective such that for each chosen answer the result is 0 or some real number. The amount of choices is restricted to nine by the current development state of the parser. In further

releases of the software this number could be changed. The single choice quiestion looks pritty *classical*:

Some question?

○ Option 1

◉ Option 2

○ Option 3

The evaluation function for a *single choice – multiple answer* question type is defined as:

$$f: A \to B$$
$$where\ A = \{\, x \in \mathbb{N} \mid x \leq 9\}\ and\ B \in \mathbb{R}$$

Obviously the argument and the function could be related in different ways. Here each choice is correct. Also it is possible to valuate each answer by any real number.

Multiple choice – multiple answers is the sum *S* of the values of each choice:

$$S = \sum_{i=1}^{9} b_i a_i$$
$$where\ a \in \mathbb{R}\ \text{and}\ b = \{0,1\}$$

The Boolean variable b represents a test whether a given choice is checked or not thus offering 2^i discrete real values for evaluation of a question. A typical multiple choice quiestion looks as follows:

Some question?

☑ Option 1

☐ Option 2

☑ Option 3

The valuation of an exact number is defined as:

$$f: \mathbb{R} \to \{0, r\}$$
$$where\ r \in \mathbb{R}$$

Thus the answer is 0 or some real number. This is apropriate for evaluation of free numerical answers of quantitative problems. The answer options are represented by single field where the number is typed.

The last evaluation option is trivially named a *target*. It resembles a discrete function with symmetric intervals around a central point. The answer is a single real number which is passed as an argument of the function:

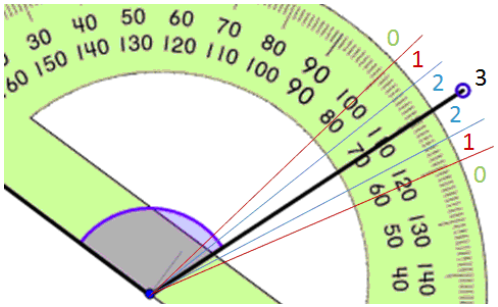
$$f: \mathbb{R} \to \mathbb{R}$$

$$\mathrm{A} = \{a_0, \dots, a_k\},\ a \in \mathbb{R} \text{ is a finite set of valuations,}$$

$$\mathrm{M} = \{m_{-k}, \dots, m_0, \dots, m_k\},\ m \in \mathbb{R} - \text{set of interval boundaries}$$

$$f(x) = \begin{cases} a_0, & x = m_0 \\ a_k, & x < m_{-k+1} \wedge x \geq m_{-k} \vee x > m_{k-1} \wedge x \leq m_k \end{cases}$$

Each interval is defined as a predicate of its boundary values. No limitation is put on the number or size of the intervas. The interval function may be depicted as follows:



4 Dynamic models

Dynamic mathematical models included as part of the problem formulations are the most important didactical novelty. They are created by means of GeoGebra. Further models are prepared for web based usage as JavaScript code. All models are collective for the entire test system and could be reused in different quiestionaries.

Preliminary qualitative survey confirms the assumption that interaction with a mathematical model will increase the comprehension of the problems thus improving the test results. Further study and analysis should be carried out to clarify qualitative aspect of integration of dynamic models in the testing process.

5 Conclusions

The system was tested in several online mathematical competitions with duration from one hour to one month. Curently the most attended contests are the issues of *Viva Mathematics with a computer* [2, 5, 6] with about of thousand participants solving up to ten problems for an hour. Bottle neck analisys reveals that the narrow segment is the database engine. The cause are numerous requests from client browsers where answers of each individual questions are recorded in real time. This requirement is mandatory since no answer should be lost if the data flow is broken on the path to the server. Therefore two options exist to increase the system productivity: optimizaion of the database behaviour and hardware enhancements.

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